OUTCOME-2

Analyze
transient and steady state response
of circuits with
DC and AC sources

Steady State Analysis of AC Circuits :

Objectives :

- Establish the phase relationship between voltage and current in a purely resistive, purely inductive, and purely capacitive circuits.
- Explain the concept of active and reactive powers.
- Make steady state analysis of *R–L* series circuits and *R–C* series circuits.
- Make steady state analysis of *R–L–C* series circuits and establish phasor relationship between voltage drops and current.
- Use phasor algebra in solving AC circuit problems.
- Make steady state analysis of AC parallel circuits.
- Solve numerical problems on AC series–parallel circuits.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

An electric circuit may consist of resistance, inductance and capacitance connected in series and parallel combinations. The circuit may have any number of such components. Such circuits will have a source of an AC supply.
For example, we may have a resistance and an inductance connected in series across a source of supply voltage. Such a circuit will be called an *R–L* series circuit.

When a resistance, an inductance and a capacitance are connected in series across a supply source, the circuit is called an *R–L–C* series circuit. Similarly, we may have parallel combination of connections of circuit elements.

We may have to calculate the current in each branch of a circuit and the total current, the power factor, the power consumed by each branch, etc.
First, we will discuss the behavior in terms of voltage, current, power and power factor of resistance, inductance and capacitance in AC circuit independently, and then discuss *R–L*, *R–C* and *R–L–C* circuits.

2.1 AC voltage applied across a resistor

Fig 2.1 shows a resistor *R* connected across a source of AC supply. The Figure also shows the wave forms of voltage, current and power consumed.
Let the supply voltage be $v = V_m \sin \omega t$.
Current $i = v/R = (V_m \sin wt)/R = (V_m/R) \sin wt$

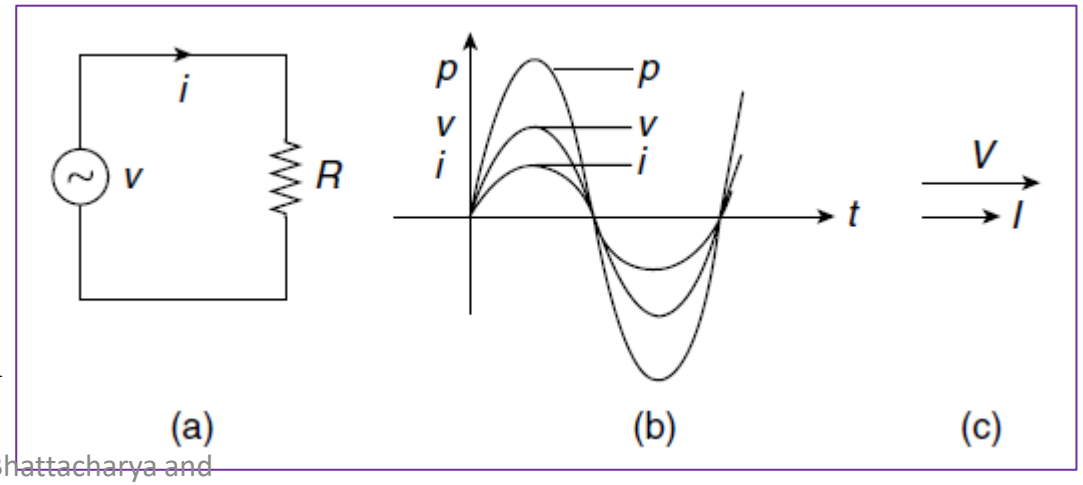


Fig 2.1 Alternating Voltage Applied Across a Resistor
(a) Circuit Diagram; (b) Wave Shapes of v, i and p and (c) Phasor Diagram

$i = I_m \sin \omega t$

where $I_m = V_m/R$

The rms value of current i is designated in capital letter as I and $I = (I_m)/\sqrt{2}$

As shown in Fig 2.1(b), both voltage and current are in time-phase, that is, there is no phase difference between the voltage and the current.

In Fig 2.1(c), the rms values of voltage and current have been shown and they are shown to be in phase.

Power is the product of voltage and current at every instant of time. The product of n and i have been calculated and has been shown. The average value of the power in a purely resistive circuit can also be calculated as follows:

$$P=1/2\Pi\int_0^{2\Pi} vid(\omega t) =1/2\Pi\int_0^{2\Pi} v_m\, sinwt\, i_m sin\omega t\, d(\omega t) =$$

$$=1/2\Pi\int_0^{2\Pi} v_m\, sin\theta\, imsin\theta d(\omega t)$$ (expressing ωt in radians, that is, substituting $\omega t = \theta$)

$$=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi} 2sin^2\theta\, d(\theta)$$

$$=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi} (1-cos2\theta)d(\theta) \quad [\therefore sin^2\theta = \frac{(1-cos2\theta)}{2}]$$

$$=\frac{V_m I_m}{4\Pi}[\theta - \frac{(sin2\theta)}{2}]_0^{2\Pi} =\frac{V_m I_m}{4\Pi}[(2\Pi\text{-}0)\text{-}0]=\frac{V_m I_m}{2}=\frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}=VI$$

The power factor is the cosine of the phase angle between the voltage and the current. Since the voltage and current are in phase, the value of power factor angle øis equal to zero.

Therefore, the power factor in a purely resistive circuit is = cos ø= cos 0° = 1.

2.2 AC voltage applied across an inductor:

In this case, let us assume a pure inductor. A pure inductor is one whose resistance is assumed to be negligible. Let such an inductor be connected across a voltage source, $v = V\text{m} \sin wt$. As a result of the application of the voltage, an alternating current i will flow through the inductor coil. This alternating current will produce an alternating magnetic field around the inductor coil.This alternating magnetic field will induce an emf in the coil which is given as follows:

$$e = L = di/dt$$

where L is the inductance of the coil and di/dt is the rate of change of the alternating current flowing through the inductor coil as shown in Fig 2.2

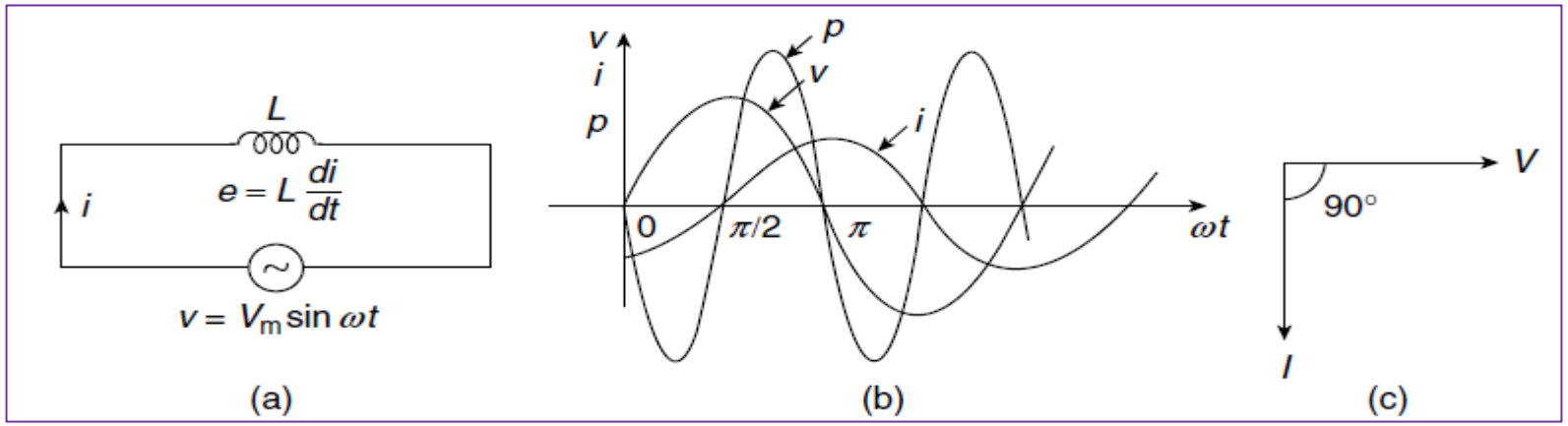


Fig 2.2 Alternating Voltage Applied Across an Inductor
(a) Circuit Diagram;
(b) Wave Shapes of Voltage, Current and Power and
(c) Phasor Diagram

This induced emf, according to Lenz's law, will oppose the voltage applied. Therefore, we can write the equation as in the following: v-=e =L $\frac{di}{dt}$

L di=v dt =Vm sinwt dt = Vm/L sinwt dt

By integrating the equation, we get the following form: i= Vm/L $\int sinwt\, dt$ = (Vm/ωL) (-coswt)

Or = (Vm/ωL) sin ($wt - \frac{\pi}{2}$)

Or = I_m sin ($wt - \frac{\pi}{2}$)

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Where $I_m=V_m/\omega L$

Thus, we observe that in a purely inductive circuit, v can be written as in the following

$v=V_m \sin \omega t$ $\quad i=I_m (\sin \omega t-\frac{\pi}{2})=I_m (\sin \omega t-90^0)$

Further, current *i* is sinusoidal but lagging behind *v* by 90°. The voltage and current wave shapes have been shown in Figure 2.2(b). The instantaneous power *p* is the product of *v* and *i*. The wave shape of instantaneous power has also been shown in the Fig.

The phasor diagram of rms values of *v* and *i* has been shown in Figure 2.2(c). In a purely inductive circuit, current *I* lags the voltage *V* by $\frac{\pi}{2}$ degrees, that is, 90°.

Power factor $\cos\theta = \cos 90° = 0$

$$P=1/2\Pi\int_0^{2\Pi} V_m \, sinwt \, I_m sin(\omega t-\frac{\pi}{2})d(\omega t)= \frac{V_m I_m}{2\Pi}\int_0^{2\Pi} sinwt \, sin\left(\omega t-\frac{\pi}{2}\right)d(\omega t)=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi} 2sinwt \, sin(\omega t-\frac{\pi}{2})d(\omega t)$$

$$=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi}\{\cos\left(w\,t-\omega t+\frac{\pi}{2}\right)-cos\left(w\,t+\omega t-\frac{\pi}{2}\right)\}d(\omega t)$$

$$=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi}\{\cos\left(\frac{\pi}{2}\right)-cos\left(2wt-\frac{\pi}{2}\right)\}d(\omega t)$$

$$=\frac{V_m I_m}{4\Pi}\int_0^{2\Pi}\{0-sin2\omega t)d(\omega t)$$

$$=\frac{V_m I_m}{4\Pi}[\frac{\cos 2\omega t}{2}]_0^{2\pi}$$

$$\frac{V_m I_m}{4\Pi}[\frac{\cos 4\pi-cos0}{2}]=0 \qquad [\therefore \cos 4\pi-cos0=1]$$

Average power in a purely inductive circuit P = 0. Hence, the average power absorbed by a pure inductor is zero.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

We had earlier taken Im=Vm/ωL
The opposition to current is ω L. This is called inductive reactance XL.
The value of XL = ω L = 2Π f L.
The opposition offered by an inductor to the flow of current is XL, that is, equal to ω L.
This is called the inductive reactance and is expressed in ohms. Inductance L is expressed in henry.
As mentioned earlier, the values of alternating quantities are expressed in terms of their effective or rms values rather than their maximum values.

Therefore, $Im=Vm/\omega L$ can be written as $Im/\sqrt{2}=\frac{Vm/\sqrt{2}}{\omega L}$

Or $I=V/X_L$, i.e $V=IX_L$

If V is taken as the reference axis, we can represent V as a phasor and can represent it as V∠0°.
Since current I is lagging voltage V by 90°,
we represent the current as I∠-90° or -jI for a purely inductive circuit.
Again, if I is taken as the reference axis, then I and V can be represented as I∠0° and V∠+90° or +jV, respectively, as shown in Fig 2.3.
Note that j is an operator that indicates the rotation of a phasor by 90° in the anti-clockwise direction from the reference axis.

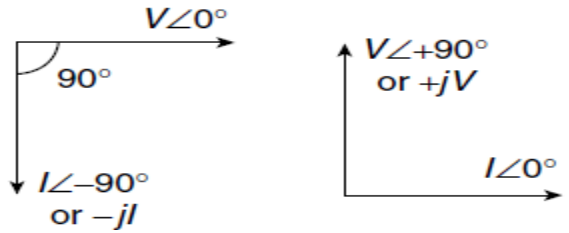


Figure 2.3 Phasor Diagram of V and I in a Purely Inductive Circuit

Concept of Reactive Power:

Let us examine why the power absorbed by a pure inductive circuit is zero. Let us refer to the inductive circuit shown in Fig 2.2. In Fig 2.2(b), it is observed that for one-half cycle, the power is negative and for the next half cycle, the power is positive. The average value of the power for a complete cycle, that is, the power consumed is zero. The positive power indicates that the power is drawn by the circuit from the supply source.

When the current rises in the circuit, energy is utilized in establishing a magnetic field around the inductor coil. This energy is supplied by the source and is stored in the magnetic field. As the current starts reducing, the magnetic field collapses and the same stored energy is returned to the supply source. Thus, in one half cycle, the power is drawn by the inductor and in the second half cycle, the power is returned to the source. In this way the net power absorbed by the inductor over a complete cycle is zero. The power, that is, being circulated from the source to the inductor and back to the source is called the reactive power. The active and reactive power will also be discussed in a separate section.

2.3 AC Voltage Applied Across A Capacitor:

A sinusoidal voltage source has been shown connected across a pure capacitor in Fig 2.4(a). When current starts flowing, the capacitor starts getting charged. Charge q of the capacitor in terms of capacitance of the capacitor C and supply voltage v is expressed as in the following: q = Cv

Current i is the rate of flow of charge. Therefore, $i = \frac{dq}{dt} = C\frac{dV}{dt} = C\frac{dV_m \sin\omega t}{dt} = \omega C\, V_m \cos\omega t$

Or $i = \omega C\, V_m \sin\left(\omega t + \frac{\pi}{2}\right)$ or $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$ Where $I_m = \omega C V_m = V_m/(1/\omega C) = V_m/X_c$ where $Xc = 1/\omega C$

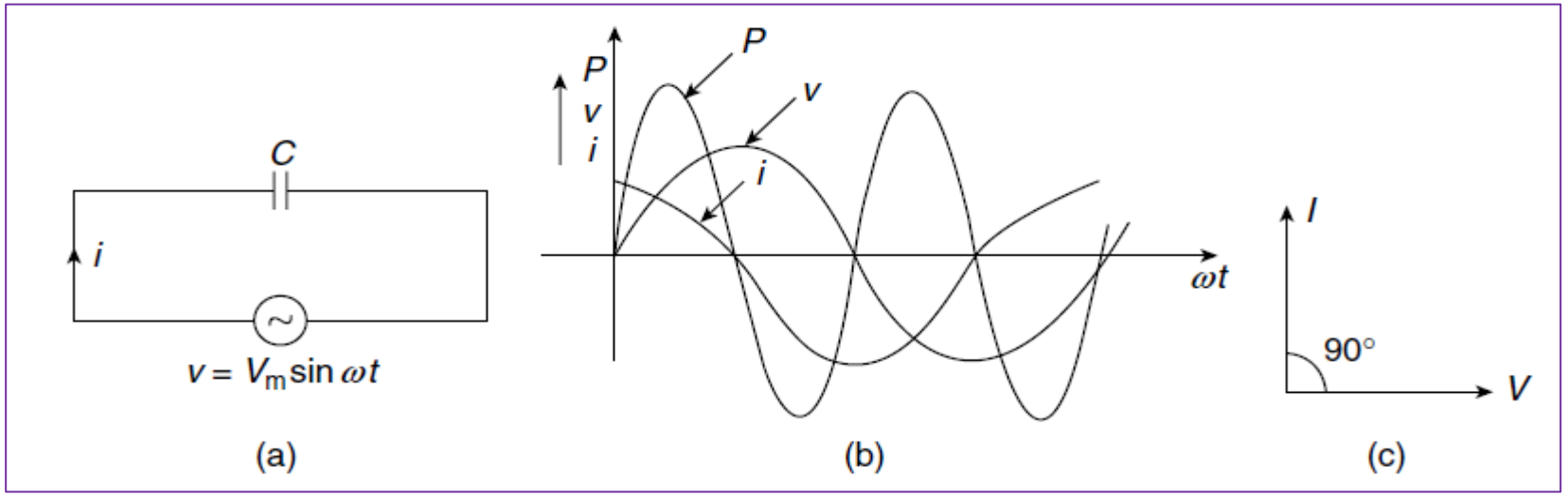


Fig 2.4 Alternating Voltage Applied Across an Capacitor (a) Circuit Diagram; (b) Wave Shapes of Voltage, Current and Power and (c) Phasor Diagram

Hence, in a pure capacitive circuit, $v = V_m \sin\omega t$ and current $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$

Thus in a purely capacitive circuit, the current leads the voltage by 90°. Since the current leads the voltage by 90°, the power factor of the purely capacitive circuit is P.f = $\cos\theta = \cos 90° = 0$

Xc = 1/ ωC is called the capacitive reactance of the capacitor.

To express in terms of rms values,

we assume $I_m/\sqrt{2}=I$ $V_m/\sqrt{2}=V$

Therefore, $V_m/X_c=I_m$ or $I=V/X_c$

The term *X*c is the opposition offered by the capacitor to the flow of current and is called capacitive reactance.

Concept of Reactive Power:

In a capacitor, similar to an inductor, the average power absorbed for a complete cycle is zero. When voltage is applied, the capacitor starts getting charged, energy gets stored in the capacitor in the form of electro-static fi eld. When the applied voltage starts falling from its maximum value, the energy starts getting returned to the supply. In this way, the power is absorbed from

and then returned to the supply source. The net power absorbed by a pure capacitor is zero. The power, that is, being circulated from the source to the capacitor and back to the source is called the reactive power.

The average or net power in a pure capacitor circuit can be calculated as given in the following:

Average power $P = \frac{1}{2\pi}\int_0^{2\pi} P\, d(\omega t) = \int_0^{2\pi} \frac{1}{2\pi} vi\, d(\omega t)$

$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(\omega t)\, I_m \sin\left(\omega t + \frac{\pi}{2}\right) d(\omega t)$

$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 2\sin\omega t \sin\left(\omega t + \frac{\pi}{2}\right) d(\omega t)$

$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \left\{\cos\left(\omega t - \omega t - \frac{\pi}{2}\right) - \cos\left(\omega t + \omega t + \frac{\pi}{2}\right)\right\} d(\omega t)$

$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \left\{\cos\left(-\frac{\pi}{2}\right) - \cos\left(2\omega t + \frac{\pi}{2}\right)\right\} d(\omega t)$

$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\omega t\, d(\omega t)$

$= 0$

Average power in a purely capacitive circuit $P = 0$.
Therefore, it is proved that the average power absorbed by a pure inductor and a pure capacitor
is zero.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

3.4 *R–L* SERIES CIRCUIT

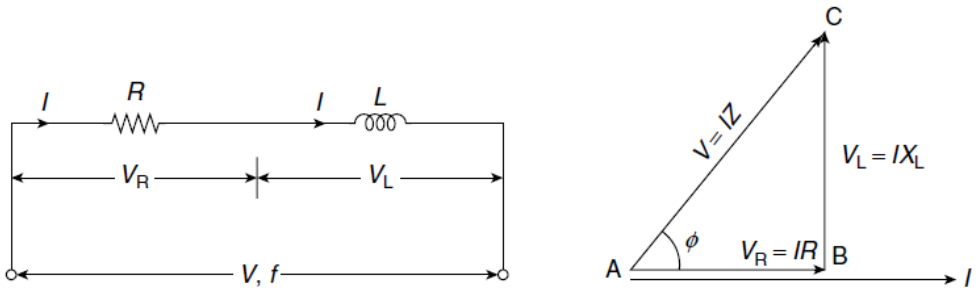
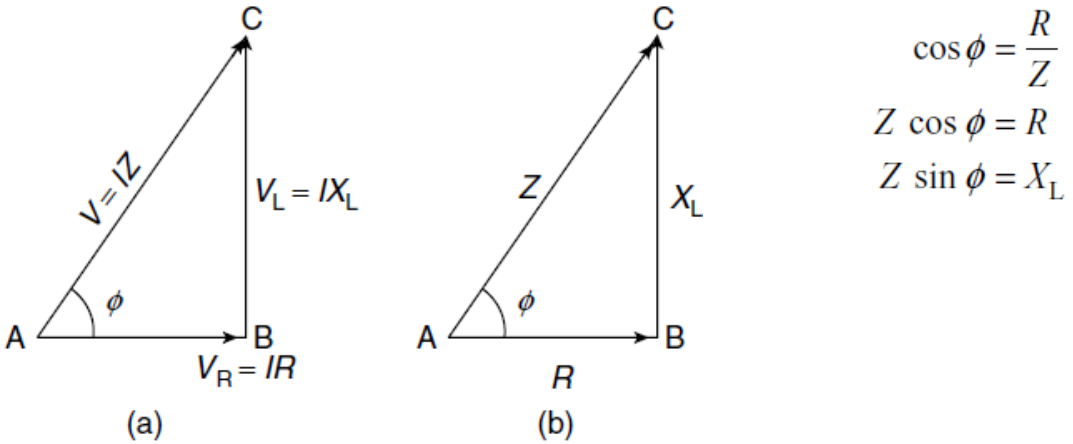


Figure 3 *R–L Series Circuit (a) Circuit Diagram and (b) Phasor Diagram*

$$V_R = IR, V_L = IX_L \text{ where } X_L = \omega L = 2\pi fL$$



$$\cos\phi = \frac{R}{Z}$$

$$Z\cos\phi = R$$

$$Z\sin\phi = X_L$$

$$V^2 = V_R^2 + V_L^2$$

or $$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

or $$V = I\sqrt{R^2 + X_L^2}$$

or $$I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z} \text{ or } V = IZ$$

where $$Z = \sqrt{R^2 + X_L^2}$$

$$Z = R + jX_L \text{ and } \cos\phi = \frac{R}{Z}$$

$$\tan\phi = \frac{X_L}{R}$$

$$\text{Power} = VI\cos\phi$$

3.5 Apparent Power, Real Power AND Reactive Power

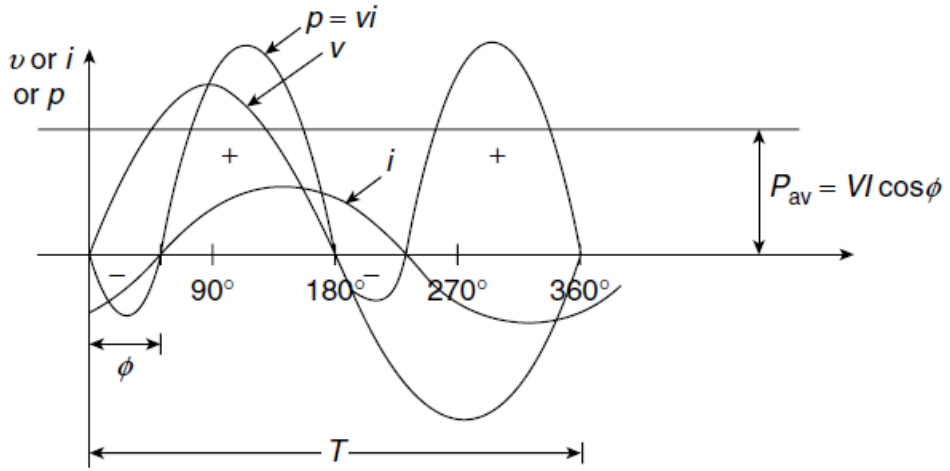
$$S^2 = P^2 + Q^2$$

$$S = \sqrt{P^2 + Q^2}$$

$$\text{KVA} = \sqrt{(\text{KW})^2 + (\text{KVAR})^2}$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

POWER IN *R–L* SERIES CIRCUIT



$$\text{kVA}\cos\phi = \text{kW}$$
$$\text{kVA}\sin\phi = \text{kVAR}$$

Fig *Wave Forms of Voltage, Current and Power in an R–L Series Circuit*

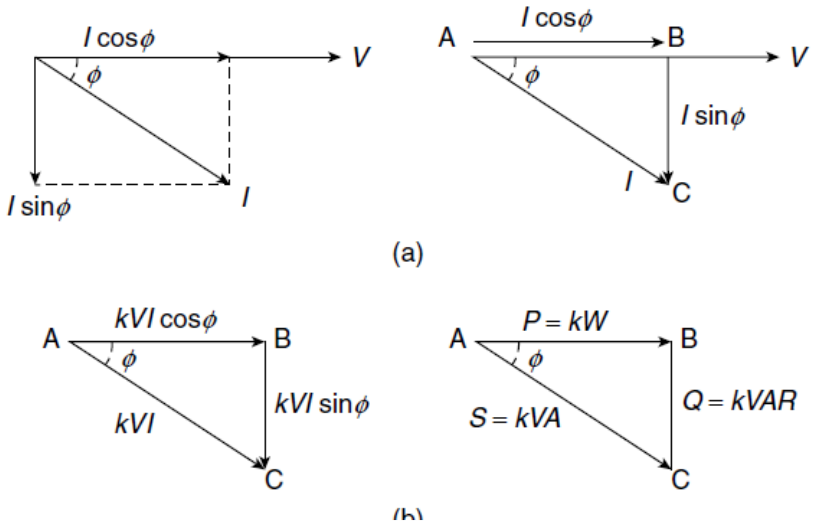


Fig. *Power Triangle Diagram Developed from Voltage–current Relationship*
(a) Voltage–current Relationship and (b) Power Triangle

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

3.8 *R–C* SERIES CIRCUIT

When the circuit draws current I, then there are two voltage drops.

1. Drop across pure resistance $V_R = I \times R$.
2. Drop across pure capacitance $V_C = I \times X_C$.

Here, $X_C = \dfrac{1}{2\pi fC}$ and I, V_R and V_C are the rms values.

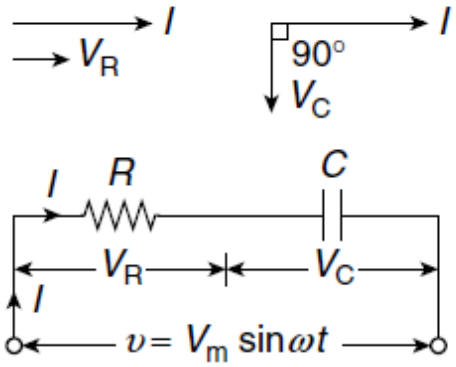


Fig *R–C Series Circuit*

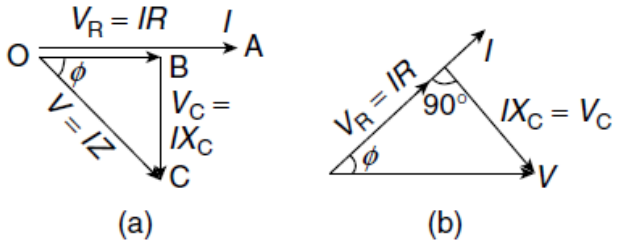


Fig. *Phasor Diagrams of R–C Circuit (a) Current, I as the Reference Phasor and (b) Voltage, V as the Reference Phasor*

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\phi = \tan^{-1}\frac{X_C}{R}$$

Applied voltage $V = \sqrt{V_R^2 + V_C^2}$

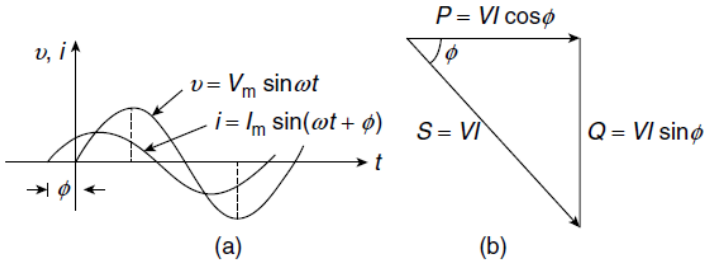
$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$V = IZ$$

$$Z = \sqrt{R^2 + X_C^2} = \text{impedance of the circuit}$$

Apparent power $S = VI$ volt amperes or VA;
Active power $P = VI\cos\phi$ W;
Reactive power $Q = VI\sin\phi$ VAR;
where $\cos\phi$ = power factor of the circuit.



Fig*(a) Wave Forms of the Voltage and the Current and Their Phase Relationship in an R–C Series Circuit and (b) Power Triangle Diagram*

3.9 *R–L–C* SERIES CIRCUIT

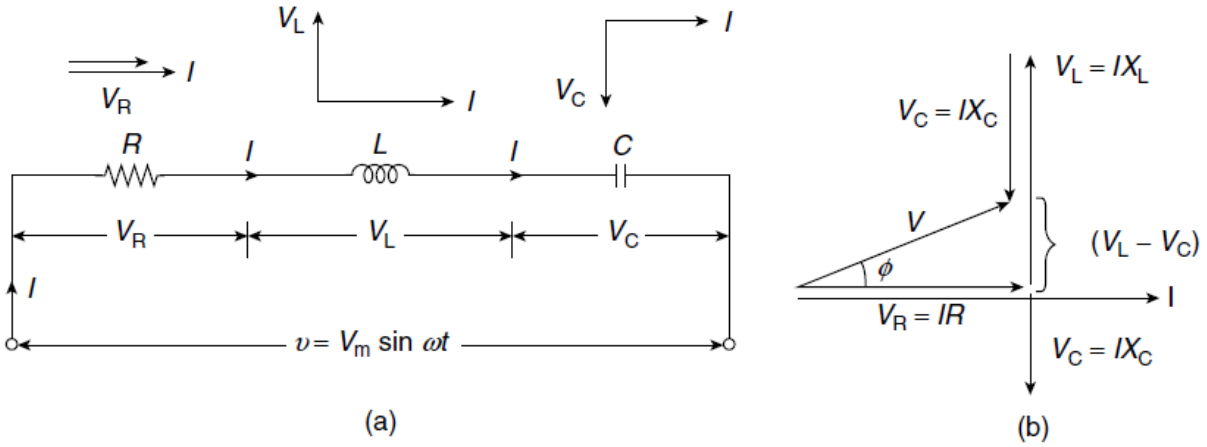
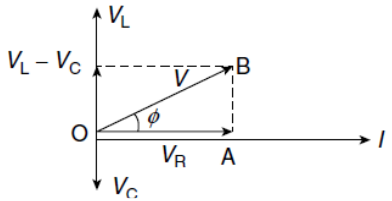
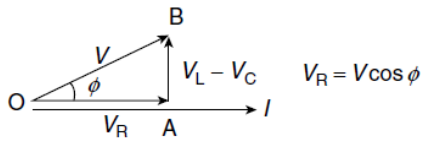


Fig *(a) R–L–C Series Circuit and (b) Phasor Diagram*

CASE 1. When $X_L > X_C$,





$V_R = V\cos\phi$

Applied voltage is,

$$OB = \sqrt{OA^2 + AB^2}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

or $$V = IZ$$

where $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{V_L V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}, \phi = \tan^{-1}\frac{(X_L - X_C)}{R}$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

CASE 2. When $X_L < X_C$,

$$\text{Applied voltage represented by OB} = \sqrt{OA^2 + AB^2}$$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{R^2 + (X_C - X_L)^2}$$

or $$V = IZ$$

where $$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{I(X_C - X_L)}{IR} = \frac{X_C - X_L}{R} \quad \therefore \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

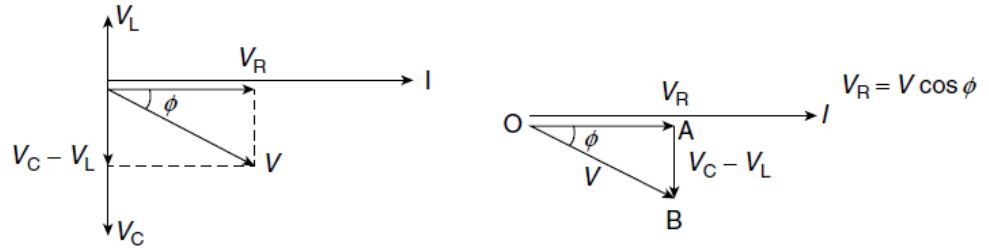


Fig *Phasor Diagram of an R–L–C Series Circuit When* $X_L < X_C$

CASE 3. When $X_L = X_C$, then $V_L = V_C$.

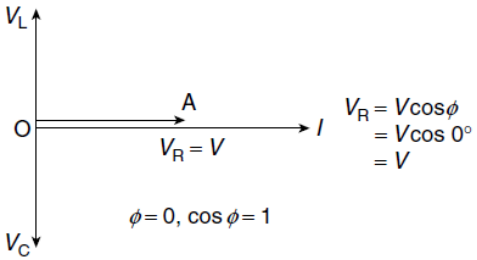


Fig *Phasor Diagram of an R–L–C Series Circuit When* $X_L = X_C$

Example 1 An inductor of 0.5 H is connected across a 230 V, 50 Hz supply. Write the equations for instantaneous values of voltage and current.

Example 2 A 230 V, 50 Hz sinusoidal supply is connected across the following: (a) A resistance of 25 $\wedge$; (b) An inductance of 0.5 H; (c) A capacitance of 100 μF. Write the expressions for instantaneous current in each case.

Example 3 An alternating voltage of rms value 100 V, and frequency 50 Hz is applied separately across a resistor of 10 $\wedge$, an inductor of 100 mH and a capacitor of 100 μF. Calculate the current fl ow in each case. Draw the phasor diagrams for each case and explain the phasor diagrams.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Transient Response of Circuits Using Differential Equations

- Explain transient condition that may occur in a network.
- Explain transient condition in a *R–L* series circuit.
- Derive expressions for rise and decay of current in a *R–L* series circuit under transient condition.
- Explain the meaning of time-constant of *R–L* series circuit.
- Solve numerical problems of *R–L* séries circuit Under transient condition.
- Explain transient response of *R–C* series circuit.
- Derive expression for charging current of a capacitor and a resistor in series on application of a DC voltage.
- Make a complete analysis of transient condition of *R–C* series circuit on sudden application of a DC voltage.
- Distinguish between steady state condition and transient condition with respect to voltage and current in a *R–L* and *R–C* series circuit with DC excitation.
- Derive expressions for current and voltage under transient condition in a*R–L–C* series circuit on sudden application of a DC excitation.
- Solve numerical on transient *R–L–C* series circuits.
- Carryout sinusoidal response i.e., response with a sinusoidally varying input in a *R–L* series circuit and *R–C* series circuit.
- Analyze transient condition in a *R–L–C* series circuit with sinusoidal input.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.1 TRANSIENT CONDITION IN NETWORKS

Electrical networks contain resistors, inductors, and capacitors. Inductors and capacitors are energy storing devices. Energy stored in these circuit elements cannot change instantaneously.

The period of adjustment during which the stored energy in these elements' changes from their initial level to the final level is called the settling time, which is generally a fraction of a second. The time taken by a circuit to change from one steady state condition to another steady state condition is called transient time.

A transient condition in networks occurs due to switching operations. During the transient period, the current and voltages change from their initial values to the new values. When a circuit is switched on, there exist two sources of energy in the circuit. One source is the initial stored energy in inductances and capacitances at the time of switching. The second energy source is the energy, that is, applied externally in the form of voltage or current sources.

The complete response of a circuit representing a system can be represented in two parts, namely, its forced response or steady state response and transient response. The transient response or solution shows the way the circuit responds when a forcing function (a voltage) as input changes in energy state. Transient response depends upon the circuit parameters and their initial charge condition.

Transient response of electrical circuits can be determined either by using differential equations or by using Laplace transform. In this chapter, we will determine transient response of *R–L*, *R–C*, and *R–L–C* circuits using differential equations.

2.2 TRANSIENT RESPONSE OF *R–L* SERIES CIRCUITS HAVING DC EXCITATION

We will consider the transient response of *R–L* circuit, *R–C* circuit and *R–L–C* circuit one by one due to the switching on DC voltage or sinusoidal voltage.

2.2.1 Rise of Current Through *R–L* Series Circuit

Let us consider that DC voltage *V* is switched on to the series *R–L* circuit at $t = 0$, as shown in Figure 2.1.Let $i(t)$ be the current fl owing through the circuit after closing the switch. Let us find an expression for current $i(t)$ after closing switch *k* at $t = 0$.

Applying KVL to the series *R–L* circuit, we get the following:

$$V = Ri + L\frac{di}{dt}$$

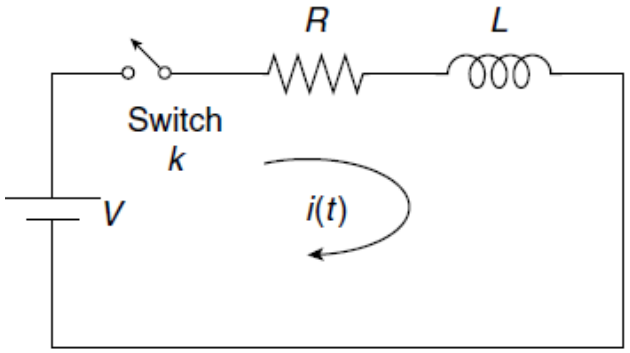


Fig 2.1 *Transient Response of R–L Circuit*

Dividing both sides by *L*, we obtain the equation as follows:

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \qquad \text{-------} \quad \text{Equation (1)}$$

Equation (1) is a first-order linear equation of the following form:

$$\frac{di}{dt} + Pi = Q$$

whose solution is given as follows:

$$i(\text{I.F}) = \int Q(\text{I.F})dt + C$$

where I.F is integrating factor, and it can be written in the following form:

$$\text{I.F} = e^{\int P dt}$$

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Now, on considering equation (1), we obtain the equation as follows:

$$\frac{di}{dt}+\left(\frac{R}{L}\right)i=\left(\frac{V}{L}\right)$$

Where $P=\frac{R}{L}$ and $Q=\frac{V}{L}$

Therefore, $\text{I.F}=e^{\int Pdt}=e^{\int\frac{R}{L}dt}=e^{\frac{R}{L}t}$

Further, its solution will be written as follows:

$$i(\text{I.F})=\int Q(\text{I.F})dt+C$$

or
$$i\left(e^{\frac{R}{L}t}\right)=\int\frac{V}{L}\cdot e^{\frac{R}{L}t}\cdot dt+C$$

$$=\frac{V}{L}\int e^{\frac{R}{L}t}dt+C$$

$$=\frac{V}{L}\cdot\frac{e^{\frac{R}{L}t}}{\frac{R}{L}}+C$$

or
$$i\left(e^{\frac{R}{L}t}\right)=\frac{V}{R}\cdot e^{\frac{R}{L}t}+C$$

or
$$i=\frac{V}{R}+\frac{C}{e^{\frac{R}{L}t}}$$

$$i=\frac{V}{R}+Ce^{-\frac{R}{L}t}$$

----------2

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Now, in order to find the value of C, we will apply the initial condition. Since, the inductor does not allow sudden change of current through it, at $t = 0$, $i = 0$

Applying the initial condition, at $\left.\begin{matrix} t=0 \\ i=0 \end{matrix}\right\}$ in equation (2), we get the equation as follows:

$$0 = \frac{V}{R} + Ce^{0}$$

or

$$0 = \frac{V}{R} + C$$

or

$$C = -\frac{V}{R}$$

Now, substituting this value of C in equation (2), we can write the equation as in the following:

$$i = \frac{V}{R} - \frac{V}{R} e^{-\frac{R}{L}t}$$

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

----------3

This gives an expression for the rise in current i through an inductor when DC voltage is applied to it. This expression has two parts, the steady state part V/R and the transient part $\frac{V}{R} e^{-\frac{R}{L}t}$.

The graphical representation of changing current in R–L circuit is shown in Fig2. Steady-state current or i_{ss} is the final current that remains in the circuit when the transient dies out. The time taken for the current to reach its steady-state value from its initial value is called transient time.

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and steady state current, $i_{ss} = \underset{t\to\infty}{\text{Lt}}\ i(t)$

or
$$i_{ss} = \underset{t\to\infty}{\text{Lt}} \frac{V}{R}\left[1 - e^{\frac{-R}{L}t}\right]$$
$$= \frac{V}{R}\left[1 - e^{-\infty}\right] = \frac{V}{R}(1-0) = \frac{V}{R}$$

Therefore, the steady-state current in *R–L* series circuit, $i_{ss} = \frac{V}{R}$.

The voltage across the resister, $V_R = Ri = R \times \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

or
$$V_R = V\left(1 - e^{-\frac{R}{L}t}\right)$$

The voltage across the inductor $V_L = L\frac{di}{dt} = L\frac{V}{R} \times \frac{R}{L}e^{-\frac{R}{L}t}$

or
$$V_L = Ve^{-\frac{R}{L}t}$$

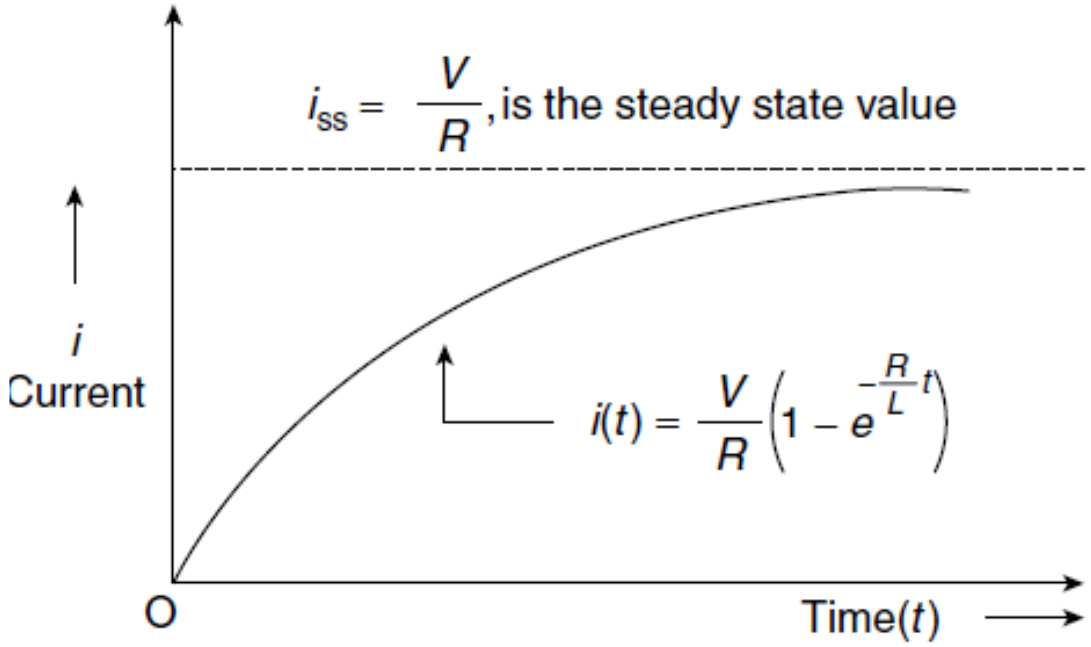

Fig 2 *Graphical Representation of Transient Current in R–L Circuit*

2.2 Time Constant of *R–L* Series Circuit

The ratio of *L* and *R* in the *R–L* circuit is called time constant. Let *t* is represented by τ Where $\tau = \frac{L}{R}$

Now, substituting $t = \tau = \frac{L}{R} =$ time constant in equation (3), we get the following

$$i = \frac{V}{R}\left[1 - e^{-\frac{R}{L}\frac{L}{R}}\right]$$

$$= \frac{V}{R}\left[1 - e^{-1}\right]$$

or $$i = \frac{V}{R}\left[1 - \frac{1}{e}\right]$$

$$i = \frac{V}{R}[1 - .368]$$

$$= 0.632\frac{V}{R} = 63.2\% \text{ of } \frac{V}{R}$$

= 63 2. %of final current, that is, the steady-state current.

Therefore, the time constant of *R–L* series circuit may be defined as the time at which the current through the *R–L* series circuit rises to 63.2% of the steady-state value. The transient part of the current reaches 36.8% of its initial value.

2.3 Decay of Current Through *R–L* Series Circuit

Let us consider that after closing the switch, the current in the circuit has reached the steady state value. Now, suddenly the voltage is disconnected by opening switch *K* and connecting it to K_1 as shown in Fig 2.3. At $t = 0$, Current *i* has been equal to V/R

.

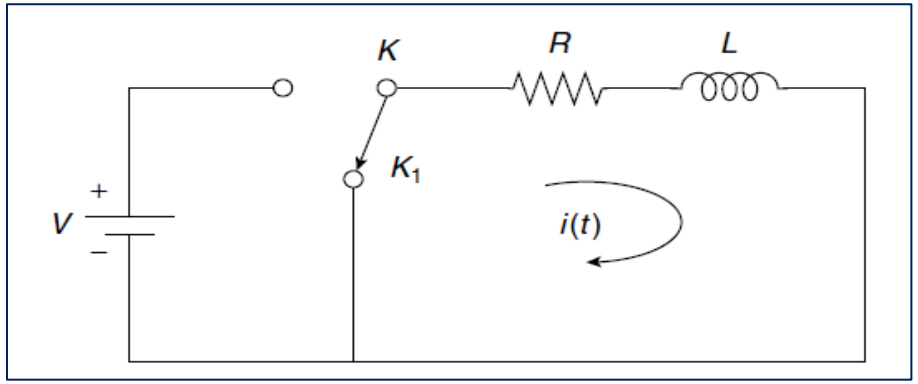

Fig 2.3 *Decay of Current in an R–L Circuit*

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Now, let us find the expression for current $i(t)$ as in the following.

By applying KVL, we obtain the equation as follows:

$$Ri + L\frac{di}{dt} = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

By comparing this equation with the form, $\frac{di}{dt} + Pi = Q$

We have $P = \frac{R}{L}$, and $Q = 0$

Therefore, $\text{I.F} = e^{\int P dt} = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$ and its solution will be given as follows

$$i(\text{I.F}) = \int Q(\text{I.F}) dt + C$$

or

$$i\left(e^{\frac{R}{L}t}\right) = O + C$$

or

$$i = \frac{C}{e^{\frac{R}{L}t}}$$

$$i(t) = Ce^{-\frac{R}{L}t}$$

----------4

Now applying the initial condition
that is, at $t = 0,\ i = \frac{V}{R}$

Substituting the value in equation (4), we get $\frac{V}{R} = Ce^0$

or $C=V/R$, By substituting this value in equation (4), we can obtain the following form:

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$$i(t) = \frac{V}{R}e^{-\frac{R}{L}t}$$

----------5

This is the expression for decay in current through the *R–L* series circuit when it is suddenly disconnected from supply. This shows that the current decays exponentially. Now, when $t = \tau = \frac{L}{R}$ (time constant), then

$$i(t) = \frac{V}{R} e^{-\frac{R}{L}\frac{L}{R}}$$

$$= \frac{V}{R} e^{-1}$$

$$= \frac{V}{R} (.368)$$

$i(t) = 36.8\% \frac{V}{R} = 36.8\%$ of initial current.

The time constant of *R–L* series circuit may also be defined as the time at which the current in the *R–L* series circuit decays to 36.8% of the initial value. The graphical representation of decay of current through *R–L* series circuit shown in Fig 2.4.
Steady-state current after the decay of current in *R–L* series circuit is calculated as follows:

$$i_{ss} = \underset{t \to \infty}{\text{Lt}}\ i(t)$$

$$= \underset{t \to \infty}{\text{Lt}}\ \frac{V}{R} e^{-\frac{R}{L}t}$$

$$= \frac{V}{R} e^{-\infty} = 0$$

This shows that under steady state condition, the current in the circuit becomes zero.

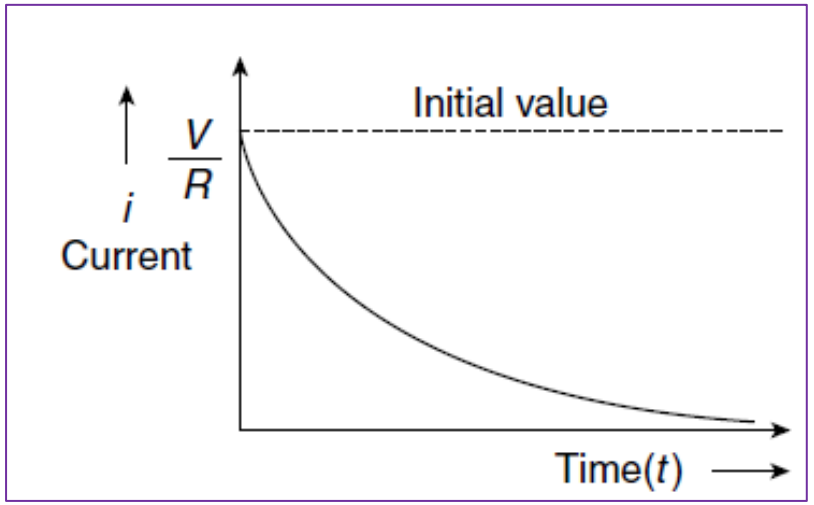

Fig 2.4 *Decay of Current in an R–L Series Circuit*

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Example 1 The values of R and L in a series R–L circuit are 10 and 40 H, respectively. At the instant of closing the switch, the current rises at the rate of 5 A/s. Calculate the following: 1. the value of applied voltage 2. rate of growth of current when 6A flows in the circuit 3. find the energy stored in the inductor.

Example 2 A DC voltage of 20 V is applied in an R–L circuit where $R = 5$ and $L = 10$ H. Calculate (a) the current i; (b) voltage across resistor and voltage across the inductor; (c) the time constant and (d) the maximum value of stored energy.

Example 3 A 100 V DC source is applied across the coil of $R = 10$ and $L = 10$ H. Find the energy supplied to the coil in the first 5 s. If the coil is disconnected and immediately short circuited ,then find the energy dissipated.

Examples 4 A coil has resistance of 1 and an inductance of 1 H. It is suddenly connected to 6 V DC voltage source. Calculate the following: 1. Initial and final values of current 2. Time constant 3. Rate of change of current at $t = 0$ and $t =$
4. Voltage across inductance at $t = 0$ and $t =$ 5. Voltage across resistance at $t = 0$ and $t =$ and 6. Current at $t = 1$ s

Example 5 A coil having a resistance of 100 and an inductance of 20 H is connected to a 200 V DC source. Suddenly, the coil is disconnected from the battery and short-circuited. Calculate the following: 1. The current in the coil at $t = 0$
2. Rate of change of current at $t = 0$ 3. Time constant and 4. Time taken for the current to decrease to 0.25A

Example 6 The fi eld winding of a DC motor has 80 resistance and 12 H inductance. It is connected to a 240 V DC source. At $t = 0$, the supply is disconnected and a fi eld discharge resistance of 80 is connected across the fi eld winding. Find the rate of change of current in the winding at $t = 0$.

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2.3 TRANSIENT RESPONSE IN *R–C* SERIES CIRCUITS HAVING DC EXCITATION

Consider an *R–C* series circuit, as shown in Fig 2.5.Let us consider two cases, that is, when the capacitor is getting charged and when the capacitor is getting discharged.

2.3.1 Case I: Capacitor is Getting Charged

When switch *K* is closed, the supply voltage *V* gets connected to the *R–C* series circuit at $t = 0$. Let us assume that initially the capacitor is fully discharged. Therefore, at $t = 0$, there is no charge stored in the capacitor. The capacitor can be assumed as short circuit initially, and hence the current fl owing in the series *R–C* circuit is *V/R*..

Thus, in a series *R–C* circuit, when initially the circuit is switched on to DC voltage source, the current flowing in the circuit is *V/R* That is, at $t = 0, i = \frac{V}{R}$.

Now, by applying KVL in the circuit of Fig 2.5, we get the following form:

$$V = Ri + \frac{1}{C}\int i\,dt$$

Differentiating both sides with respect to time, we obtain the equation as follows:

$$0 = R\frac{di}{dt} + \frac{1}{C}i$$

or

$$\frac{di}{dt} + \frac{1}{RC}i = 0$$

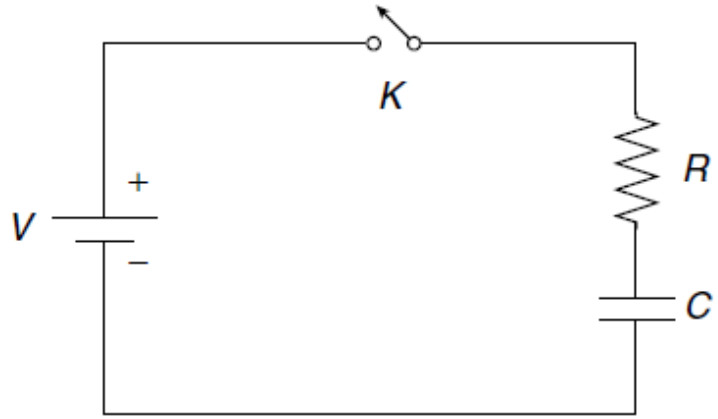

Fig 2.5 *Transient Response of R–C Series Circuit*

Comparing it with the Leibnitz equation, we can write the equation as in the following:

$$\frac{di}{dt} + Pi = Q$$

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We get the equation as follows: $P = \frac{1}{RC}$ and $Q = 0$

Therefore, integrating factor (I.F) $= e^{\int P dt} = e^{\int \frac{1}{RC} dt} = e^{\frac{1}{RC} t}$

Now, the solution of equation (7) will be given as follows:

$$i(\text{I.F}) = \int Q(\text{I.F}) dt + K_1$$

or

$$i\left(e^{\frac{t}{RC}}\right) = 0 + K_1,$$

where K_1 is the constant of integration

or

$$i = \frac{K_1}{e^{\frac{t}{RC}}}$$

$$i = K_1 e^{-\frac{t}{RC}}$$

Now, in order to find the value of K_1, apply the initial condition in the equation:

That is, substituting at $\left.\begin{matrix} t = 0 \\ i = \frac{V}{R} \end{matrix}\right\}$ in equation (8), we get $\frac{V}{R} = K_1 e^0$

or

$$K_1 = \frac{V}{R}$$

Now, substituting the value of K_1 in equation (8), we get the following form:

$$i = \frac{V}{R} e^{-\frac{t}{RC}}$$

This is the expression for the current flowing in the *R–C* circuit at any time *t*, after the switch is closed at $t = 0$. The graphical representation of response of the circuit fl ow in the *R–C* circuit is shown in Fig.11.

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Time Constant of R–C Series Circuit

$\tau = RC$ is the time constant of R–C circuit Now, when $t = \tau = RC$, then we substitute t in equation (9) and we get the following:

$$i = \frac{V}{R} e^{-\frac{RC}{RC}}$$

$$= \frac{V}{R} e^{-1}$$

$$= \frac{V}{R} \cdot \frac{1}{e}$$

$$= .368 \frac{V}{R} = \text{ 36.8\% of initial current}$$

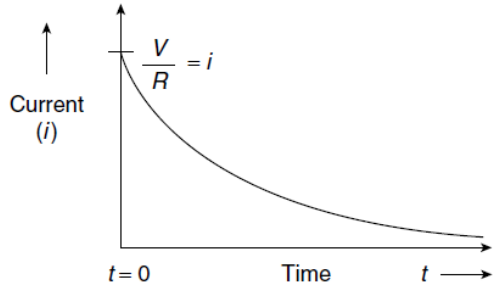

Fig 11 *Charging Current in an R–C Series Circuit*

The time constant of R–C series circuit is that time at which the current fl owing in the circuit reaches 36.8% of initial value.

Voltage Across the Capacitor

$$V_C = \frac{1}{C} \int i dt$$

By substituting the value of i from equation (9), we can write the equation as follows:

$$V_C = \frac{1}{C} \int \frac{V}{R} e^{-\frac{t}{RC}} dt$$

$$= \frac{V}{RC} \int e^{-\frac{t}{RC}} dt$$

$$= \frac{V}{RC} \cdot \frac{e^{-\frac{t}{RC}}}{\frac{-1}{RC}} + K_2$$

where K_2 is the constant of integration $V_C = -Ve^{-\frac{t}{RC}} + K_2$ --------10

Now, at $\left.\begin{matrix} t = 0 \\ V_C = 0 \end{matrix}\right\}$ we assumed the capacitor to be fully discharged initially.

Therefore, we get the following from equation (10):

$$0 = -Ve^0 + K_2$$

or $K_2 = V$

By substituting the value of K_2 in equation (10), we can write the equation as follows:

$$V_C = -Ve^{-\frac{t}{RC}} + V$$

$$V_C = V\left[1 - e^{-\frac{t}{RC}}\right] \qquad \text{----------11}$$

This is the expression for the capacitor voltage.

Equation (11) is indicating that the capacitor gets charged exponentially when a DC voltage is applied across it. The graphical representation of charging of the capacitor when a DC voltage is switched on is shown in Fig12.

It is concluded that when a series *R–C* circuit is supplied with a DC voltage, then capacitor starts charging and capacitor current starts decaying as depicted graphically in Fig13. From the graphs in Fig13, it is observed that initially $V_C = 0$ (minimum) and $i = V/R$ = (maximum).

However, with the passage of time, *i* decays exponentially and V_C rises exponentially. Finally, when capacitor charges to maximum value *V*, current *i* through the circuit becomes zero. The instantaneous values of current and capacitor voltage are as follows:

$$i = \frac{V}{R}e^{-\frac{t}{RC}}$$

$$V_C = V\left[1 - e^{-\frac{t}{RC}}\right]$$

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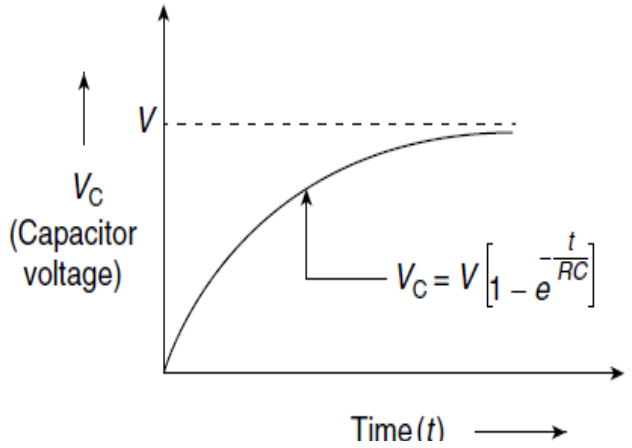

Fig12 *Gradual Building Up of Voltage Across the Capacitor*

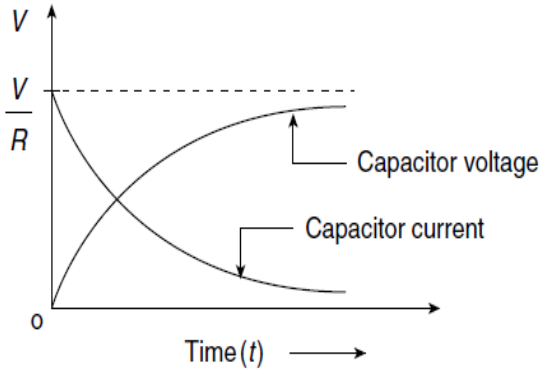

Fig13 *Graphical Representation of Capacitor Voltage and Capacitor Current*

Steady-State or Final Values

The steady-state value of current is given as follows:

$$i_{ss} = \underset{t\to\infty}{\text{Lt}}\, i(t)$$
$$= \underset{t\to\infty}{\text{Lt}} \frac{V}{R} e^{-\frac{t}{RC}}$$
$$= \frac{V}{R} e^{-\infty}$$
$$= 0$$

The steady-state value of capacitor voltage can be written as in the following:

$$V_{ss} = \underset{t\to\infty}{\text{Lt}}\, V_C(t)$$
$$= \underset{t\to\infty}{\text{Lt}}\, V\left[1 - e^{-\frac{t}{RC}}\right]$$
$$= V\left[1 - e^{-\infty}\right]$$
$$= V(1-0)$$
$$= V$$

2.3.2 Case II: Discharging of Capacitor

Now, let us consider a discharging case, that is, when the fully charged capacitor is disconnected from DC supply. Let the switch is moved to position K_1, as shown in Fig14. Now, the direction of current is opposite to that of the previous case.
Let us apply KVL, we get the following:

$$Ri + \frac{1}{C}\int i\,dt = 0$$

Differentiating the equation with respect to time, we can form the equation as follows:

$$R\frac{di}{dt} + \frac{1}{C} i = 0 \quad \text{or} \quad \frac{di}{dt} + \frac{1}{RC} i = 0 \qquad \text{----------12}$$

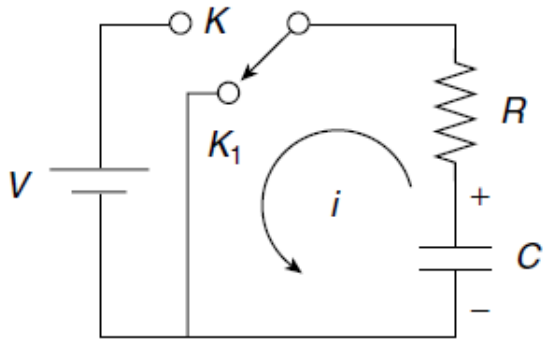

Fig14 *Discharging of the Capacitor when the Switch is Moved from Position K to K_1*

By comparing equation (12) with the Leibnitz equation, we write it in the following form

$\frac{di}{dt} + Pi = Q.$ Therefore, we get $P = \frac{1}{RC}, Q = 0$ Further, $\text{I.F} = e^{\int P dt} = e^{\int \frac{1}{RC} dt} = e^{\frac{1}{RC}t}$

and solution of equation (12) will be $i(\text{I.F}) = \int Q(\text{I.F}) dt + K_3$

where K_3 is the constant of integration

or $i \cdot e^{\frac{t}{RC}} = 0 + K_3$

or $i = K_3 e^{-\frac{t}{RC}}.$ -------13

Now, in discharging case, at $t = 0$, i=-V/R (negative sign is due to the opposite direction of current) By applying the initial condition in equation (13), we get the following:

$$-\frac{V}{R} = K_3 e^0$$

$$K_3 = -\frac{V}{R}$$

Substituting this value in equation (13), we get the following form: $i = -\frac{V}{R} e^{-\frac{t}{RC}}$ ----------14

This is the expression for transient discharging current. The graphical representation of discharging current has been shown in Fig15. The direction of discharging current has been shown as negative.

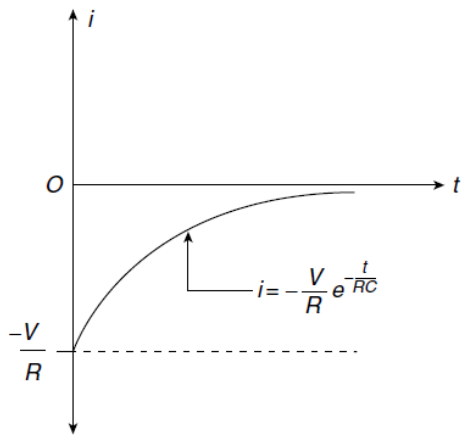

Fig15 *Discharging Transient Current in R–C Circuit*

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Voltage Across Capacitor During Discharging

The transient voltage across the capacitor during discharging is given as follows:

$$V_C = \frac{1}{C}\int i\,dt$$

$$= \frac{1}{C}\int -\frac{V}{R}e^{-\frac{t}{RC}}.dt$$

$$= -\frac{V}{RC}\cdot\frac{e^{-\frac{t}{RC}}}{-\frac{1}{RC}} + K_4$$

where K_4 is the constant of integration

Therefore, $V_C = Ve^{-\frac{t}{RC}} + K_4$ ----------15

Now, applying the initial condition, that is, at $t = 0$ and $V_C = V$ (maximum) in equation (15), we have the following.

$$V = Ve^0 + K_4$$
$$K_4 = 0.$$

By substituting the value of K_4 in equation (15), we get the form as follows:

$$V_C = Ve^{-\frac{t}{RC}}$$

The equation indicates that the capacitor voltage decays exponentially and the graphical representation of capacitor voltage during discharging has been shown in Fig16.

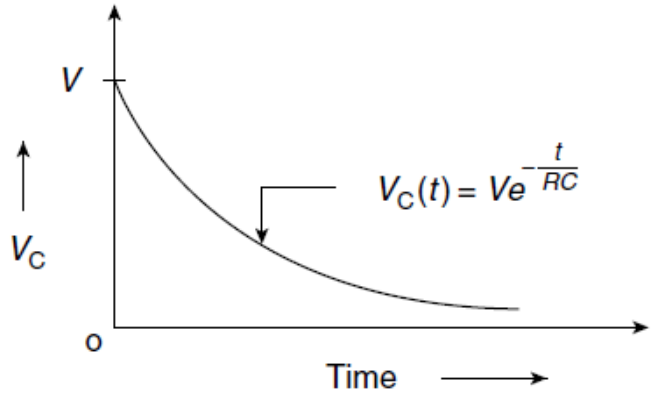


Fig16 *Capacitor Voltage During the Period of Discharging*

2.4 TRANSIENT RESPONSE OF *R–L–C* SERIES CIRCUITS HAVING DC EXCITATION

Consider an *R–L–C* series circuit as shown Fig18. Let us assume that the capacitor and inductor are initially uncharged, that is, at $t = 0$, there is no charge on L or C. Let us find the expression for current in the circuit when switch S is closed at $t = 0$. By applying KVL, we get $V = Ri + L\frac{di}{dt} + \int \frac{1}{C} i dt$

Differentiating the above equation with respect to t, we get the following:

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

Rearranging the equation, we can rewrite it as follows: $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$ ---- 16

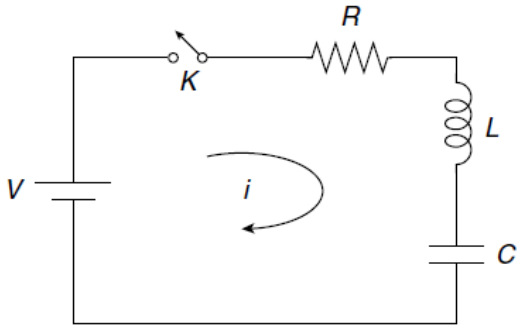


Fig18 *Transient Response of R–L–C Series Circuit*

Substituting, $\frac{d}{dt}$ for and $\frac{d^2}{dt^2}$ for D^2

We get the symbolic form of equation (16) as

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \qquad \text{----------17}$$

Now, the characteristics equation of (17) is given by the following: $D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$

The roots of characteristic equation are as follows:

$$D = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2(1)}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$= \frac{-R}{2L} + \sqrt{\frac{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}{4}}$$

$$= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Let us consider the following two cases.

Case I: when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, then roots are real and unequal and are given as follows:

$$a = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$b = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Therefore, the response is given by the following: $i = C_1 e^{at} + C_2 e^{bt}$

In this case, the overdamped response is obtained. The overdamped response is shown in Fig19.

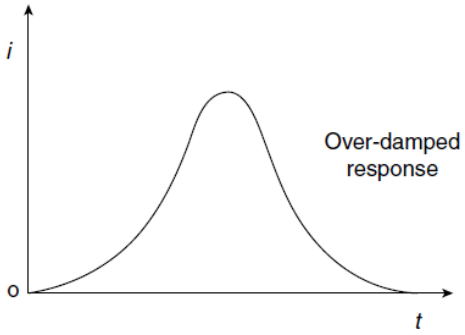


Fig 19 *Overdamped Response of Current in R–L–C Series Circuit*

Case II: when $\left(\frac{R}{2L}\right)^2 \le \frac{1}{LC}$, , then roots are complex conjugate and provide the underdamped response as shown in Fig20.

Now, $D = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

If $\left(\frac{R}{2L}\right)^2 \le \frac{1}{LC}$, then

$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}$ will be a negative quantity

$$= -\frac{R}{2L} \pm \sqrt{-\left\{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right\}}$$

$$= -\frac{R}{2L} \pm j\sqrt{\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2}$$

$$= \alpha \pm j\beta$$

Then, solution is given by the following:

$$i = e^{\alpha t}(C_1 \cos \beta t + C_2 \sin \beta t)$$

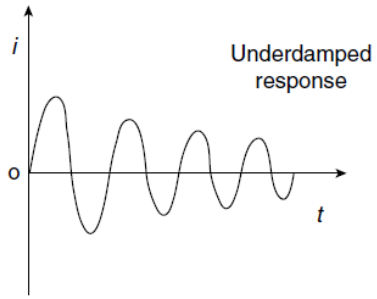


Fig 20 *Underdamped Response of Current in R–L–C Series Circuit*

Case III: when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ then roots are equal and they provide a critical damped response as shown in Fig 21.

$$D = -\frac{R}{2L}, -\frac{R}{2L}$$

Then solution will be written as in the following $i = C_1 e^{-\frac{R}{2L}t} + C_2 e^{-\frac{R}{2L}t}$

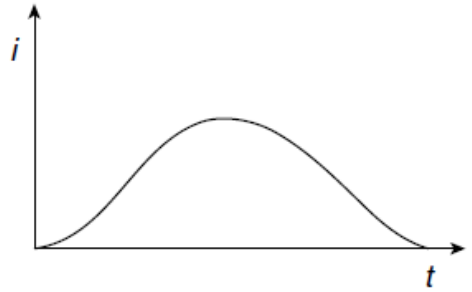


Fig 21 *Critical Damped Response of R–L–C Series Circuit*

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.5 SINUSOIDAL RESPONSE OF *R–L* CIRCUITS

We will now study the transient response of circuits of *R*, *L*, and *C* when a sinusoidally varying voltage is applied across such circuits. Let us consider a series *R–L* circuit shown in Fig 23. When the switch *S* is closed, a sinusoidal voltage $V_m \sin(\omega t + \theta)$ is applied to series *R–L* circuit. Here, θ is the phase angle and V_m is the amplitude of line voltage wave.

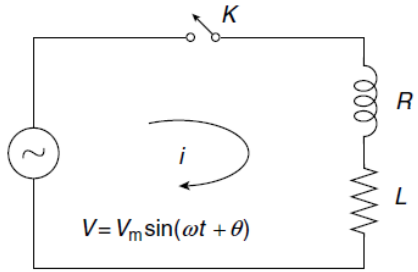


Fig 23 *Transient Response of R–L Circuit to Sinusoidal Input Voltage*

By applying KVL, we get the form as follows:

$$V_m \sin(\omega t + \theta) = Ri + L\frac{di}{dt}$$

or

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V_m}{L}\sin(\omega t + \theta)$$

Its symbolic term is given as follows:

$$\left(D + \frac{R}{L}\right)i = \frac{V_m}{L}\sin(\omega t + \theta)$$

Here, $D = \frac{d}{dt}$.

The complementary function of equation (24) can be obtained from characteristic equation. The characteristic equation of equation (24) is given as in the following:

$$D + \frac{R}{L} = 0$$

or

$$D = -\frac{R}{L}$$

Therefore, the complementary function of the solution *i* is calculated as follows: $i_C = K_1 e^{-\frac{R}{L}t}$

Particular solution: let us obtain particular solution by using the method of undetermined co-efficient. Let

$$i_P = A\cos(\omega t + \theta) + B\sin(\omega t + \theta)$$

Now $\frac{di_P}{dt} = -A\omega\sin(\omega t + \theta) + B\omega\cos(\omega t + \theta)$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

$$B = \frac{V_m \cdot R}{R^2 + \omega^2 L^2} \qquad A = \frac{-\omega L V_m}{R^2 + \omega^2 L^2} \qquad \phi = \tan^{-1}\frac{\omega L}{R}$$

$$i_P = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\left[\sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{R}\right)\right]$$

Therefore, the complete solution is as follows: $i = i_C + i_P$

$$K_1 = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i = \left[\frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin\left(\theta - \tan^{-1}\frac{\omega L}{R}\right)\right]\cdot e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}\sin\left(\omega t + \theta - \tan^{-1}\frac{\omega L}{\sqrt{R}}\right)$$

This is the final expression for current.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.6 SINUSOIDAL RESPONSE OF R-C CIRCUITS

Let us consider a series RC circuit where the input voltage is a sinusoidal one, as shown in Fig 25.Let us consider at $t = 0$, switch S is closed and sinusoidal voltage $V_m \sin(\omega t + \theta)$ is applied to the circuit. Let us find the expression for the current. By applying KVL, we get the form as follows:

$$V_m \sin(\omega t + \theta) = Ri + \frac{1}{C}\int i\,dt$$

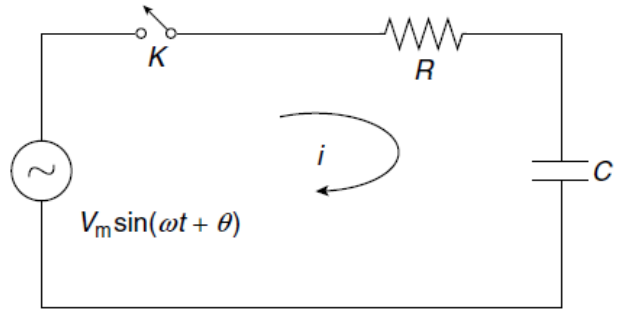


Fig 25 *Transient Response of R–C Circuit with Sinusoidal Voltage*

Differentiating the equation with respect to t, we rewrite the form as in the following:

$$V_m \omega \cos(\omega t + \theta) = R\frac{di}{dt} + \frac{1}{C}i$$

or

$$\frac{di}{dt} + \frac{1}{RC}i = \frac{V_m \omega}{R}\cos(\omega t + \theta)$$

Its characteristic equation is as follows:

$$D + \frac{1}{RC} = 0$$

or

$$D = -\frac{1}{RC}$$

Therefore, the complementary function will be given as follows:

$$i_C = K_1 e^{-\frac{1}{RC}t}$$

Now, the particular solution can be obtained by the method of undetermined coefficient.

Let

$$i_P = A\cos(\omega t + \theta) + B\sin(\omega t + \theta)$$

Now,

$$\frac{di_P}{dt} = -A\omega \sin(\omega t + \theta) + B\omega \cos(\omega t + \theta)$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

$$B = \frac{V_m \cdot R}{R^2 + (1/\omega C)^2}$$

$$A = \frac{V_m}{\omega C\left[R^2 + (1/\omega C)^2\right]}$$

$$\tan\phi = \frac{1}{\omega CR}$$

$$\text{or } \phi = \tan^{-1}\frac{1}{\omega CR}$$

$$i = i_C + i_P$$

$$i = K_1 e^{-\frac{t}{RC}} + \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \sin\left(\omega t + \theta + \tan^{-1}\frac{1}{\omega CR}\right)$$

$$K_1 = \frac{V_m}{R}\sin\theta - \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\theta + \tan^{-1}\frac{1}{\omega CR}\right)$$

$$i = \left[\frac{V_m}{R}\sin\theta - \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\theta + \tan^{-1}\frac{1}{\omega CR}\right)\right] e^{-\frac{t}{RC}}$$

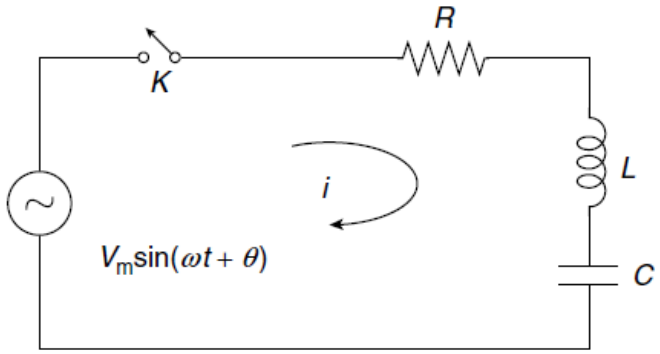
$$+ \frac{V_m}{\sqrt{R^2 + (1/\omega C)^2}} \sin\left(\omega t + \theta + \tan^{-1}\frac{1}{\omega CR}\right)$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.7 SINUSOIDAL RESPONSE OF *R–L–C* CIRCUITS

Let us consider a circuit *R–L–C* series circuit as shown in Fig27. At $t = 0$, the switch is closed, and sinusoidal voltage $V_m \sin(\omega t + \theta)$ is applied to the *R–L–C* series circuit. Applying KVL, we get the following form:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = V_m \sin(\omega t + \theta)$$



Differentiating both sides with respect to time, we obtain the equation as follows:

$$R\frac{di}{dt} + \frac{Ld^2i}{dt^2} + \frac{1}{C}i = V_m \omega \cos(\omega t + \theta)$$

or
$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{C}i = \frac{V_m \omega}{L}\cos(\omega t + \theta)$$

The corresponding characteristic equation is,

$$D^2 + \frac{R}{L}D + \frac{1}{C} = 0$$

or
$$D = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \times 1 \times \frac{1}{C}}}{2}$$

or
$$D = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{L}}$$

Let us consider the following three cases.

Case I: when $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$, then roots will real and unequal.

Therefore, $i_c = C_1 e^{at} + C_2 e^{bt}$

Case II: when $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$, roots will be complex conjugate.

Therefore, $i_c = e^{at}[C_1 \cos bt + C_2 \sin bt]$

Case III: when $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$ and $D = -\frac{R}{2L}, -\frac{R}{2L}$ (real and equal), then the following form can be obtained:

$$i_c = C_1 e^{-\frac{R}{2L}t} + tC_2 e^{-\frac{R}{2L}t}$$

$$A = \left(\frac{1-\omega^2 LC}{\omega RC}\right)B$$

$$B = \frac{V_m \cdot \frac{\omega^2 R}{L^2}}{\left(\frac{1}{LC}-\omega^2\right)^2 + \frac{\omega^2 R^2}{L^2}}$$

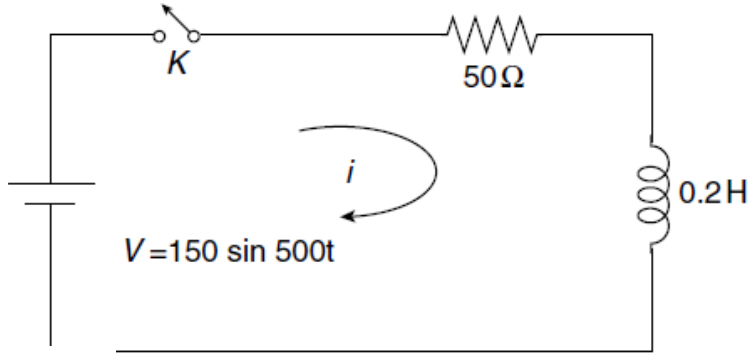
$$A = \frac{V_m \omega\left[\frac{1}{LC}-\omega^2\right]}{L\left[\left(\frac{1}{LC}-\omega^2\right)^2 + \frac{\omega^2 R^2}{L^2}\right]}$$

$$i_P = \frac{V_m}{\sqrt{\left(\frac{1}{\omega C}-\omega L\right)^2}} \times \left[\sin(\omega t+\theta)\cdot\frac{R}{\sqrt{\left(\frac{1}{\omega C}-\omega L\right)^2}} + \cos(\omega t+\theta)\cdot\frac{\left(\frac{1}{\omega C}-\omega L\right)}{\sqrt{\left(\frac{1}{\omega C-\omega L}\right)^2}}\right]$$

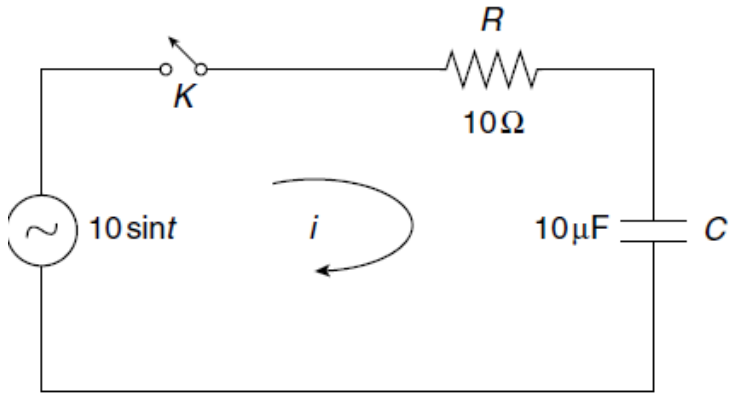
By substituting these values in the equation, we obtain the following form:
The complete solution is given as,
$i = i_c + i_P$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Example 7. A coil a resistance of 50 and an inductance of 0.2 H. Find the expression for the current if the coil is excited by a voltage 150 sin 500 *t*.

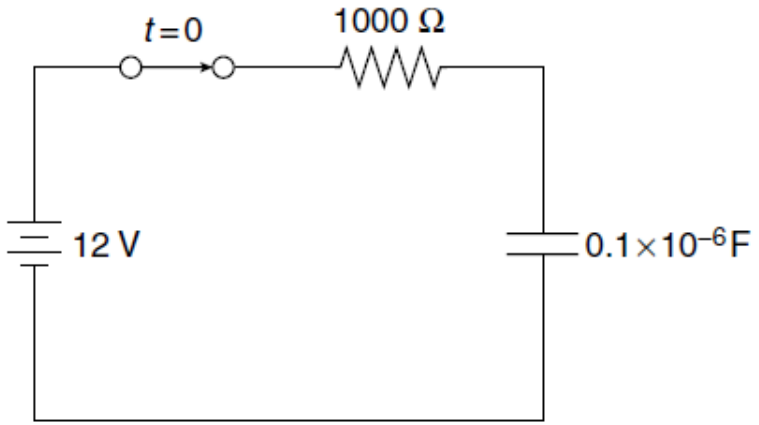


Example 8. In the circuit shown in Figure, obtain the expression for current.



Example 9 A circuit has resistance of 1000 and a series capacitance of 0.1 μF. At $t = 0$, it is connected to a 12 V battery. Find the following:

1. The current at $t = 0$
2. Rate of change of current at $t = 0$
3. Rate of change of capacitor voltage at $t = 0$
4. Current at $t = 0.1$ ms
5. Voltage across capacitor at 0.1 ms



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Transient Response of Circuits Using Laplace Transform :

OBJECTIVES:

List the steps to find transient response of electrical networks using Laplace transform.

Write differential equations of circuit variables in time domain and convert them into Laplace transform form.

Determine transient response of *R – C* circuit using Laplace transform and appreciate the method.

Determine transient response of *R – L* circuit using Laplace transform with DC excitation.

Carryout transient response analyses of *R–L–C* series circuit with DC excitation by Laplace transform method.

Solve numerical problems on transient analysis of *R–C*, *R–L*, and *R–L–C* series circuit using Laplace transform method.

Make analysis of *R–C* and *R–L* series circuits with sinusoidal input by Laplace transform method.

Solve numerical on transient analysis of circuits with sinusoidal input voltage by Laplace transform method and appreciate the method over time domain analysis method.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.1 steps to find transient response using Laplace transform

Laplace transform is a very useful mathematical tool used in determining the transient response of electrical networks.

The basic procedure in the form of steps to find the transient response using Laplace transform are mentioned in the following:

Step 1: write the differential equation of the circuit in time domain, using KVL.
Step 2: take Laplace transform of the differential equation.
Step 3: find $I(s)$ or $V(s)$.
Step 4: take the inverse Laplace transform to find $i(t)$ or $v(t)$.
We will first express the circuit elements in s-domain and then proceed to write the differential equation of circuits and take their Laplace transform.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

2.2 CIRCUIT ELEMENTS IN THE *s*-DOMAIN

2.2.1 Resistor in the *s*-Domain

Consider a resistor R in a circuit, as shown in Fig.1, where the current and voltage are shown in time domain as $i(t)$ and $v(t)$, respectively.

By ohm's Law, we get the following form: $V = iR$

Taking Laplace transform on both sides, we get the following equation:

$$L[v] = RL[i]$$

$$\text{or } V(s) = RI(s) \qquad (2.1)$$

This equation shows that the resistance element does not change while going from *t*-domain to *s*-domain.

The *s*-domain equivalent circuit of Figure 2.1 is shown in Figure 2.2.

Fig *Resistor in Time Domain*

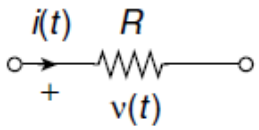
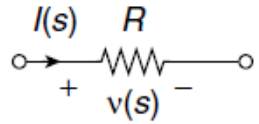


Fig *Resistor in s-Domain*



2.2.2 Inductor in *s*-Domain

Let us consider an inductor with initial current I_0 as shown in Figure 2.3. When a changing current is flowing through the inductor. We can write,

$$v = L\frac{di}{dt}$$

Taking the Laplace transform of the equation, we get the following equation: $V(s) = L[sI(s) - i(0)]$

Now, we have assumed $i(0) = I_0$

Therefore, $V(s) = L[sI(s) - I_0]$

$$V(s) = sLI(s) - LI_0$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

Equation (2.2) can be implemented using the circuit shown in Figure 2.4.
If I_0 is taken as 0, the equivalent of inductor in *s*-domain will be as shown in Figure 2.5.

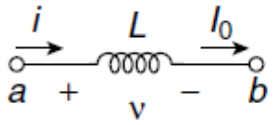


Fig *Inductor in Time Domain*

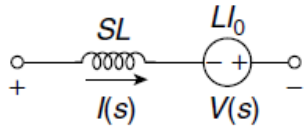


Fig *Inductor in s-Domain*

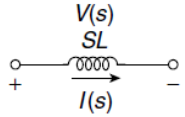


Fig *Inductor in s-Domain Under Zero Initial Condition*

2.2.3 Capacitor in *s*-Domain

Consider a capacitor in a circuit where voltage and current are expressed in *t*-domain, as shown in Fig. Let the initial voltage on the capacitor be v_0 volts Now, in *t*-domain, $i = C\frac{dv}{dt}$

Taking the Laplace transform on both sides, we get the following:

$$L\,[i(t)] = CL\left[\frac{dv}{dt}\right]$$

$$I(s) = C[sV(s) - v(0)]$$

Substitute $v(0) = V_0$

$$I(s) = C[sV(s) - V_0]$$

$$I(s) = Cs\,V(s) - C\,V_0$$

or $$V(s) = \frac{I(s)}{Cs} + \frac{V_0}{s}$$

This equation can be represented by the circuit shown in Fig. If

$V_0 = 0,$

$V(s) = \frac{1}{Cs} I(s)$

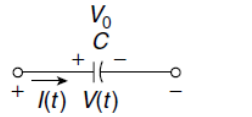


Fig *Capacitor Placed in t-Domain Circuit*

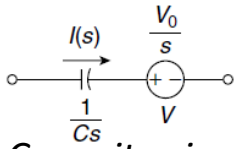


Fig *Capacitor in s-Domain*

If initial capacitor voltage is zero, that is, $V_0 = 0$, then the equivalent circuit in *s*-domain will be as shown in Fig.

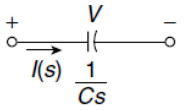
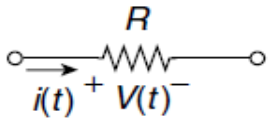
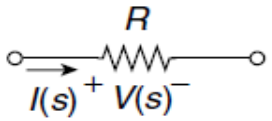
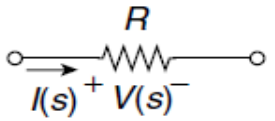
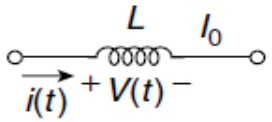
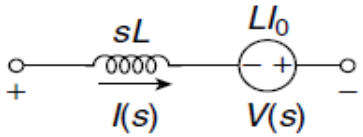
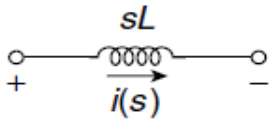
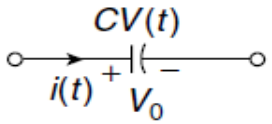
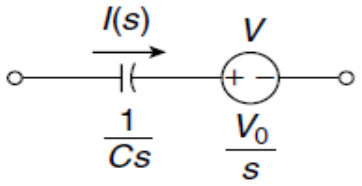
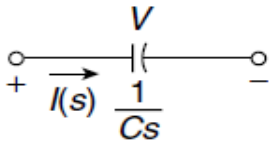


Fig *Capacitor in s-Domain when Initial Voltage is Zero*

Table 1 provides a summary of representation of circuit elements in *t*-domain and *s*-domain.

Element	*t*-Domain Circuit	*s*-Domain Circuit	*s*-Domain Equivalent Circuit (When Initial Condition is Zero)
Resistor			
Inductor			
Capacitor			

2.3 DC RESPONSE OF *R–C* SERIES CIRCUIT

$$i(t) = \frac{V}{R} e^{\frac{-t}{RC}}$$

2.4 DC RESPONSE OF *R–L* SERIES CIRCUIT

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

2.5 DC RESPONSE OF AN *R–L–C* SERIES CIRCUIT

$$i(t) = \frac{V}{L}\left[K_1 e^{-\alpha t} + K_2 e^{-\beta t}\right]$$

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh

SINUSOIDAL RESPONSE OF R–L SERIES CIRCUIT

$$A = \frac{L}{-j2\omega(R - j\omega L)} \qquad B = \frac{L}{j2\omega(R + j\omega L)} \qquad C = \frac{L^2}{R^2 + \omega^2 L^2}$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\tan^{-1}\frac{\omega L}{R}\right) e^{-\frac{R}{L}t}$$

This is the expression for the transient response.

SINUSOIDAL RESPONSE OF *R–C* SERIES CIRCUIT

$$A = \frac{RC}{2(1 - j\omega RC)} \qquad B = \frac{RC}{2(1 + j\omega RC)}$$

The value of C $\dfrac{-RC}{1 + \omega^2 R^2 C^2}$

$$\theta = \tan^{-1}\frac{1}{\omega CR}$$

or
$$i(t) = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \times \left[\sin\left(\omega t + \tan^{-1}\frac{1}{\omega CR}\right) - \sin\left(\tan^{-1}\frac{1}{\omega CR}\right) e^{-\frac{t}{RC}}\right]$$

The equation is the transient response of *R–C* series circuit to sinusoidal input.

Ref: Network Analysis and Synthesis by s. k. Bhattacharya and Manpreet singh